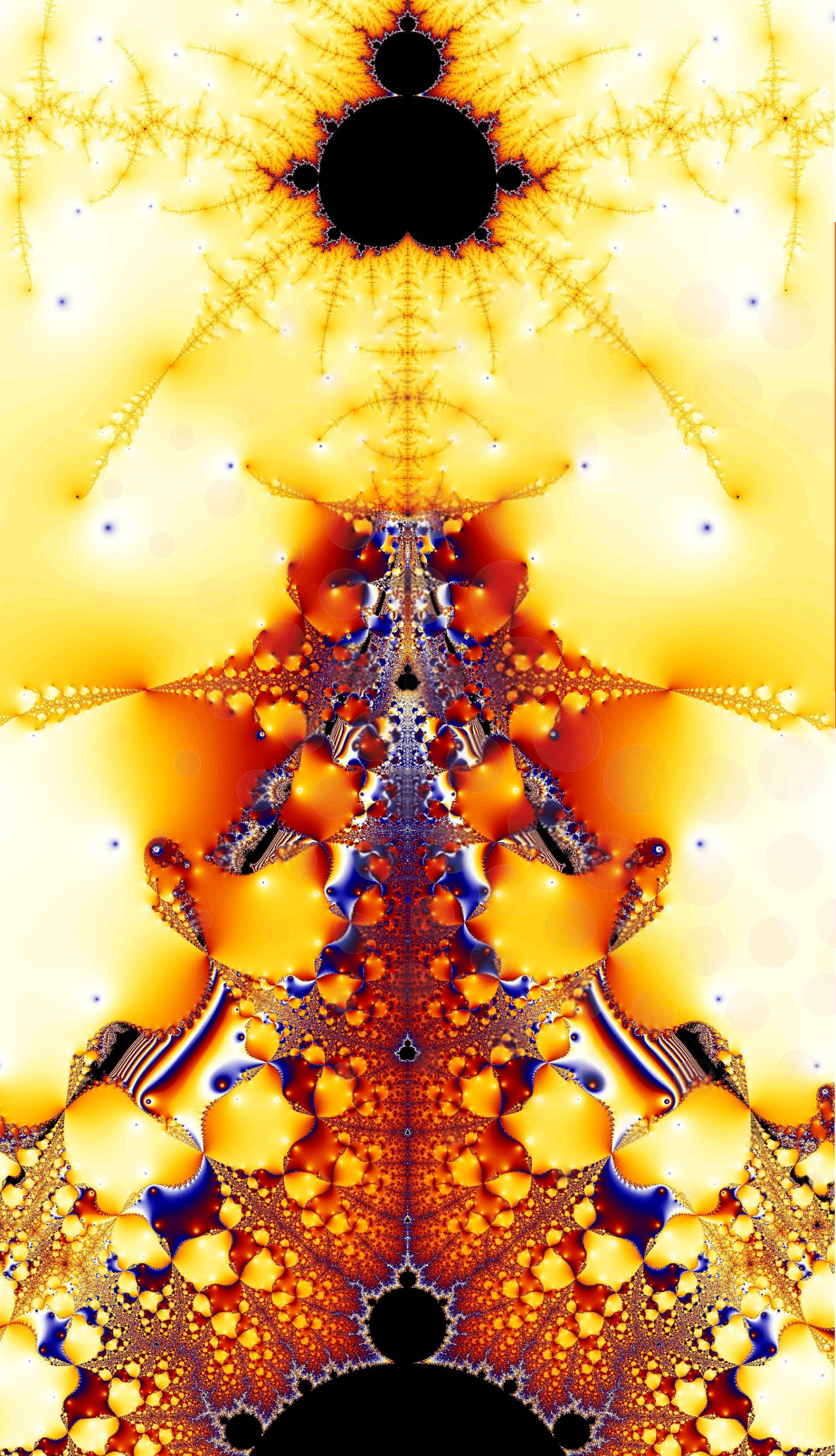


Mathematics

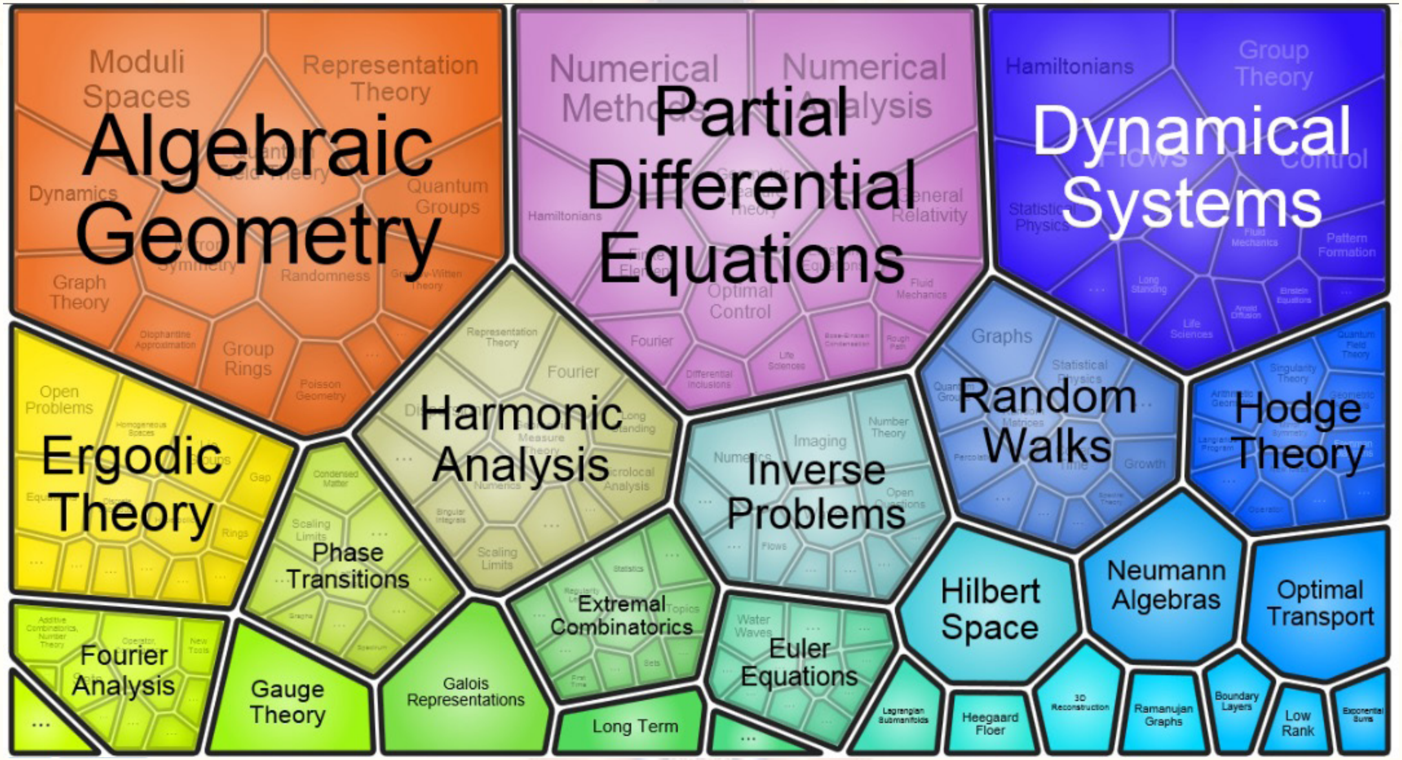
Spotlight on ERC projects

2018

European Commission

European Research Council

Established by the European Commission



Clusters of most used keywords in ERC projects related to mathematics research

The European Research Council

Set up in 2007, the European Research Council (ERC) is the first pan-European funding body designed to support investigator-driven frontier research and to stimulate scientific excellence across Europe. It aims to support the best and most creative scientists to identify and explore new directions in any field of research (Physical Sciences and Engineering, Life Sciences and Social Sciences and Humanities) with no thematic priorities and the only evaluation criterion being excellence. To date, the ERC has awarded over 8 000 long-term grants to individual researchers of any nationality and age who wish to carry out their research projects in Europe. With a budget of over EUR 13 billion from 2014 to 2020, the ERC is part of the EU Research and Innovation framework programme, Horizon 2020.

Introduction

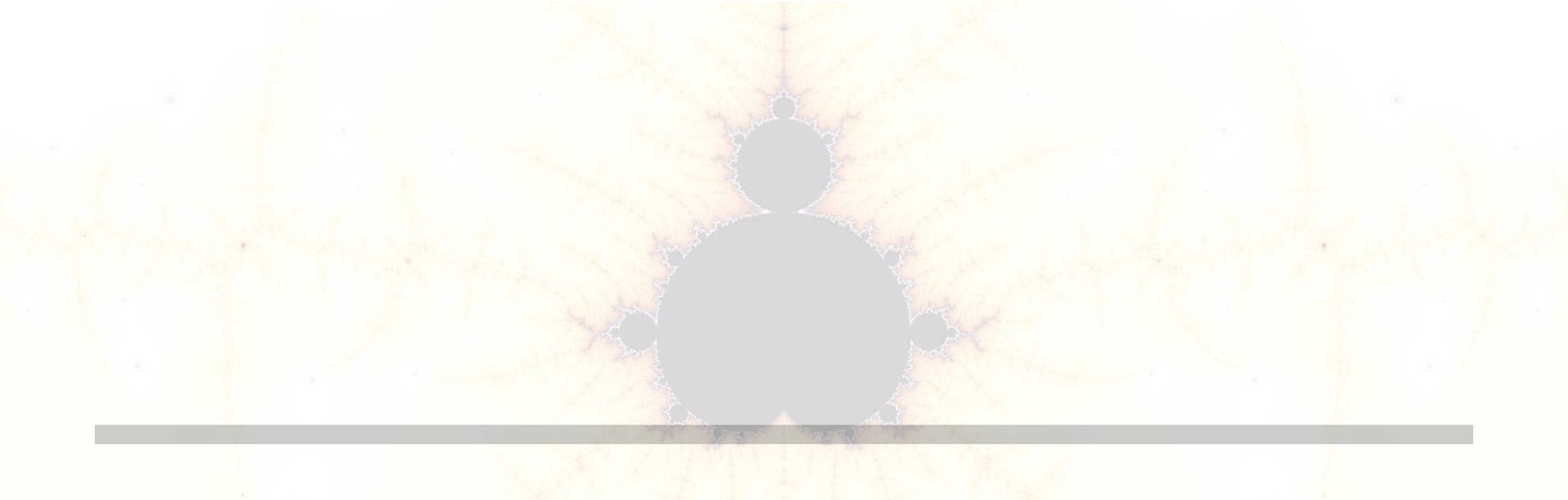
Unlike the common and extended idea that mathematics is an isolated and somehow lonely subject, research in this field, over the years, especially in recent ones, is continuously disproving this statement. Mathematics is probably much closer to real life than many other disciplines. It is playing an important role in countries' economies and is key in the current data science era.

Within the field itself and in addition to the classical interactions among different mathematical disciplines, mathematics is experiencing lately an increasing collaboration between subfields, where interaction was somehow not expected in the past; some examples being probability and combinatorics, commutative algebra and statistics or topology, geometry and analysis. The joint forces and the application of well-known tools in one subfield to another has opened up Pandora's box, giving birth in some cases to new interesting areas of research and solving/advancing in problems that were open for years. More and more, the ERC grantees build cross-disciplinary teams with very heterogeneous backgrounds where their expertise complements that of other members, being able to attack problems that would resist otherwise. Cross-disciplinarity is playing a crucial role in the recent advances in mathematics.

Similarly, mathematics has kept its traditional relationships with other fields such as computer science, engineering, econometrics or physics, since nowadays science builds highly on computation and simulation where mathematics has proven to be of great help. Lately, mathematics is more and more showing its transversal value, especially in the field of life sciences. Mathematics inspires other sciences but also gets inspired by them, some very exciting mathematical topics have originated from concepts in, for example, theoretical physics, where mathematicians have found subjects of study that hide highly rich mathematical structures.

Mathematics is growing more and more interconnected with research at the interface of multiple mathematical subfields and is at the same time increasing its connections with other sciences.

To date, the European Research Council (ERC) has supported 377 projects in mathematics, representing more than EUR 600 million of investment in this research area. A sample of these projects is highlighted in this brochure published on the occasion of the International Congress of Mathematicians 2018 (ICM 2018), taking place from 1 to 9 August in Rio de Janeiro (Brazil).



Classifying von Neumann algebras

Group theory, functional analysis and ergodic theory – three distinct areas of mathematics that meet within the theory of von Neumann algebras. The RIGIDITY project, funded by the ERC, aims to classify families of von Neumann algebras.

In mathematics, groups encode symmetry. Functional analysis, on the other hand, provides a solid mathematical framework for particle physics, since quantum mechanical observables behave like matrices of infinite size. Last but not least, ergodic theory provides us with the tools needed to understand the long-term behaviour of a dynamical system.

These three areas meet in the theory of von Neumann algebras. But how do we classify these algebras? Finding the answer to this fundamental question is exactly what the RIGIDITY project aims to do. "*Using Popa's deformation/rigidity theory, we classify von Neumann algebras that arise from group actions and free probability theory*," explains Prof. Stefaan Vaes, who leads the project. "*The ultimate goal is to understand which aspects of a symmetry group can be recovered from the much coarser structure given by the ambient von Neumann algebra*."

As to group actions, whenever a group acts on a measured space there is an associated crossed product von Neumann algebra. "*The first key result obtained in the RIGIDITY project was the construction of von Neumann algebras with precisely two crossed product decompositions*," explains Prof. Vaes.

Von Neumann algebras arise naturally in free probability theory. Very little is known about the classification of the so-called free Araki-Woods factors. This, however, is changing as Prof. Vaes and his team have established definitive classification theorems for these factors.

A third focus of the RIGIDITY project is on the quantum symmetries of von Neumann algebras. "*Our work here led us to the first truly quantum instance of Kazhdan's property (T), which is a rigidity property used in the construction of expander graphs and thus widely used in the theoretical aspects of computer science*," adds Prof. Vaes.

Researcher: Stefaan Vaes, KU Leuven (Belgium)

ERC project: Rigidity and classification of von Neumann algebras (RIGIDITY)

ERC funding: Consolidator Grant 2013, EUR 1.4 million (2014-2019)

Stefaan Vaes is a mathematician working in operator algebras and functional analysis. He leads the Functional Analysis research group at KU Leuven (Belgium). The focus of his work is on von Neumann algebras and their connections to ergodic theory, group theory and subfactors. In 2015, he was awarded the Francqui prize, the main Belgian scientific award.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New approaches to controlling dynamics

Once limited to modelling physical problems in engineering, today Partial Differential Equations (PDEs) are used by a diverse array of fields, from natural resources to meteorology, aeronautics, oil and gas and biomedicine – to name only a few. But key mathematical issues remain unsolved, particularly when addressing their control, a must in technological transfer. The ERC-funded DYCON project aims to find answers.

A PDE is a model used to describe motion: elastic bodies, fluids, crowds, opinions, etc. It is expressed as an algebraic system involving the partial derivatives of an unknown function and state (depending on space and time variables). PDEs are used to mathematically formulate and describe the motion of the relevant entities in nature and industry, such as heat and sound, fluid flow, elasticity, infections and diseases.

PDEs must be solved and, often times, in practical applications, also controlled, designed and tuned. Prof. Enrique Zuazua explains: "*Although there are a range of methods for solving and controlling PDEs, some of the key mathematical issues remain unsolved, thus limiting their applicability to real-life problems*".

Prof. Zuazua and his team of international researchers aim to make a ground-breaking contribution to the broad area of Control of PDEs. To do this, their work focuses on addressing the key unsolved analytical and computational issues in the field. "*The coordinated and focused effort that we aim to develop is timely and much needed to solve these issues and will help bridge the gap between PDE control and real applications via computer simulations*" says Prof. Zuazua.

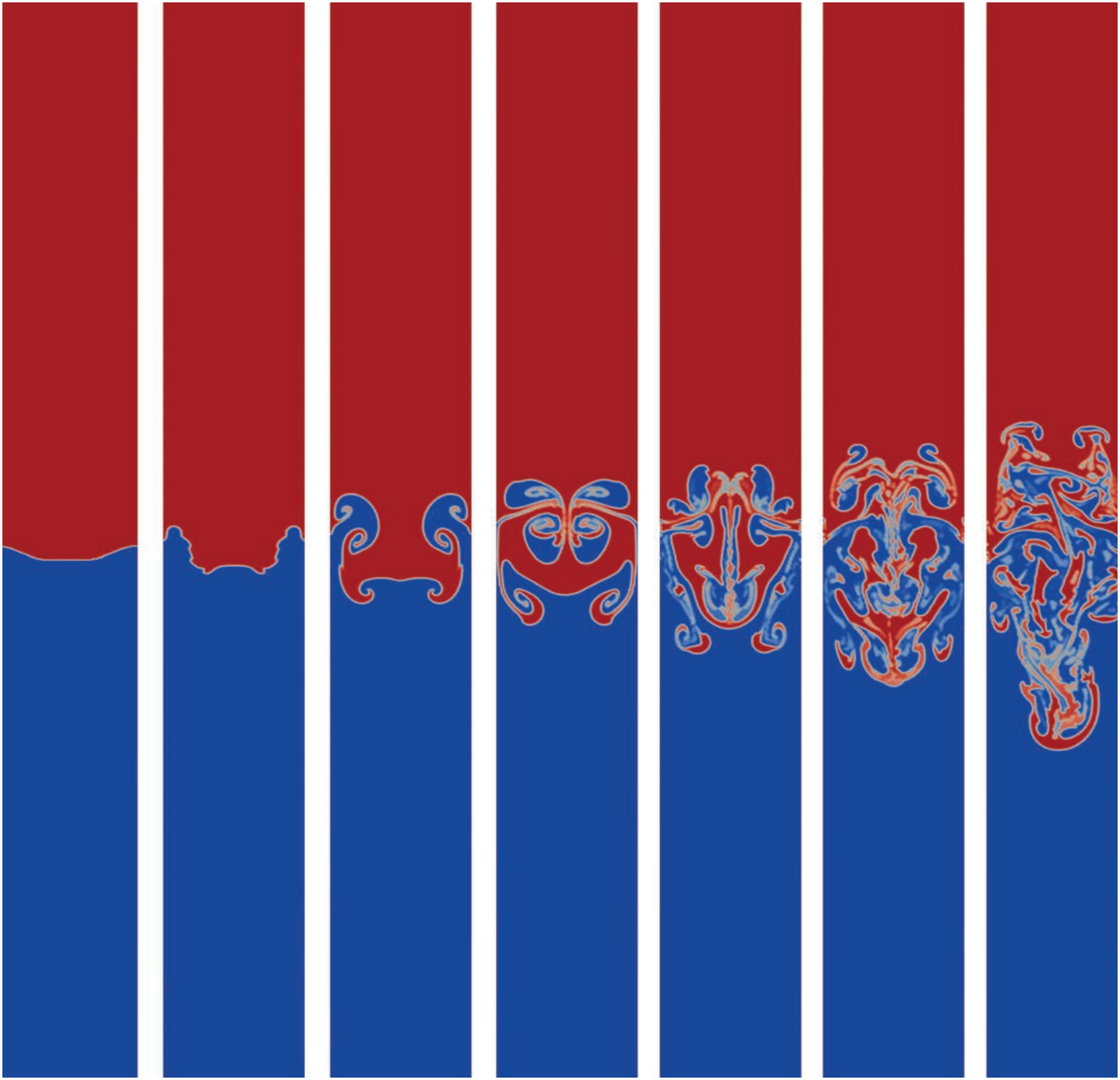
The numerous algorithms, tutorials, sample codes, software and simulations developed in the course of the project will all be made available via the DYCON Computational Laboratory. Freely available via the project webpage, these methods and tools have also been released via Zenodo, GitHub and MathWorks.

Researcher: Enrique Zuazua, Fundación Deusto (Spain)

ERC project: Dynamic Control and Numerics of Partial Differential Equations (DYCON)

ERC funding: Advanced Grant 2015, EUR 2 million (2016-2021)

Enrique Zuazua is the Director of the Chair in Computational Mathematics at DeustoTech Laboratory at the University of Deusto, Bilbao and a Professor of Applied Mathematics at the Universidad Autónoma de Madrid – UAM (Spain). His fields of expertise, in the broad area of Applied Mathematics, cover topics related with Partial Differential Equations, Systems Control and Numerical Analysis.



Rayleigh Taylor Instability

Getting to grips with (slow) chaos

Chaotic systems are everywhere: the weather, molecules in a gas, the stock market. Small variations in initial conditions can lead to a drastically different time evolution, a phenomenon known as the butterfly effect. Systems can be classified according to how fast different, nearby initial conditions diverge in time. Supported by the ERC, Prof. Corinna Ulcigrai is investigating systems for which nearby initial conditions diverge slowly in time, to uncover mechanisms which explain their complex behaviour.

The ChaParDyn project is focussing on so-called parabolic dynamical systems: mathematical models for the many phenomena which display a "slow" form of chaotic evolution. Examples of parabolic systems span from famous physics models, like the Novikov model describing electrons in metals, to fundamental mathematical objects, such as flows on surfaces.

Prof. Ulcigrai, who leads the project, explains, "*When it comes to parabolic dynamical systems, the problem is that only few examples are understood well and have been studied in depth. Each example tends to display different chaotic features, so it's hard to identify common phenomena and mechanisms for chaos.*"

Many of the parabolic systems we know best have a form of 'homogeneity', and in some sense have a lot of internal structure. One of the project components is to study 'perturbations' of this system, where this homogeneity and structure breaks down. For these 'inhomogeneous' systems, one cannot use algebraic tools, but some geometric features still persist and can be exploited.

While fundamental research, such as that carried out by Prof. Ulcigrai, is not driven by potential applications of results, virtually all of the systems that impact our everyday lives are chaotic, so 'understanding chaos' mathematically is essential. The project's research highlights the central role played by geometric phenomena in explaining the features of 'slow chaos'. The project also demonstrated that when homogeneity is broken, typical features do indeed seem to appear, which gives hope for a unifying description.

Researcher: Corinna Ulcigrai, University of Bristol (United Kingdom)

ERC project: Chaos in Parabolic Dynamics: Mixing, Rigidity, Spectra (ChaParDyn)

ERC funding: Starting Grant 2013, EUR 1.2 million (2014-2019)

Corinna Ulcigrai studied at the Scuola Normale in Pisa (Italy) and received her PhD from Princeton University under Y. Sinai in 2007. She became Full Professor at the University of Bristol in 2015 and at the University of Zurich in 2018. Her awards include a European Mathematical Society Prize (2012), a Whitehead Prize (2013) and a Wolfson Research Merit award (2017).

New tools to understand the large-scale behaviour of complex systems

Understanding complex structures means separating irrelevant information to get to something simpler and easier to understand. When you look at something from a distance – although you don't see all the details, you can still describe what you see. ERC grantee Balázs Szegedy has developed several mathematical tools for providing a compressed yet useful view of complex structures.

When it comes to understanding complex systems, the challenge is to separate the signal from the noise. For example, to describe how water flows, you don't need to 'see' the position of every single molecule, a course, large-scale view is often enough to predict the behaviour of the system.

"*To understand complex systems, we usually don't need a complete description of all the parts,*" says Prof. Szegedy. "*The challenge is to separate the relevant information, known as the structure, from the irrelevant noise.*"

To help researchers make this separation, the StrucLim project developed a variety of mathematical tools for compressing data describing large, complex systems into a simpler, more practical form. Many of these tools are based on the emerging subject of graph limit theory. "*An important achievement of the project is that we were able to connect random matrix theory with graph limit theory through our results on random regular graph*," explains Prof. Szegedy. "*It is always exciting to find connections between different fields.*"

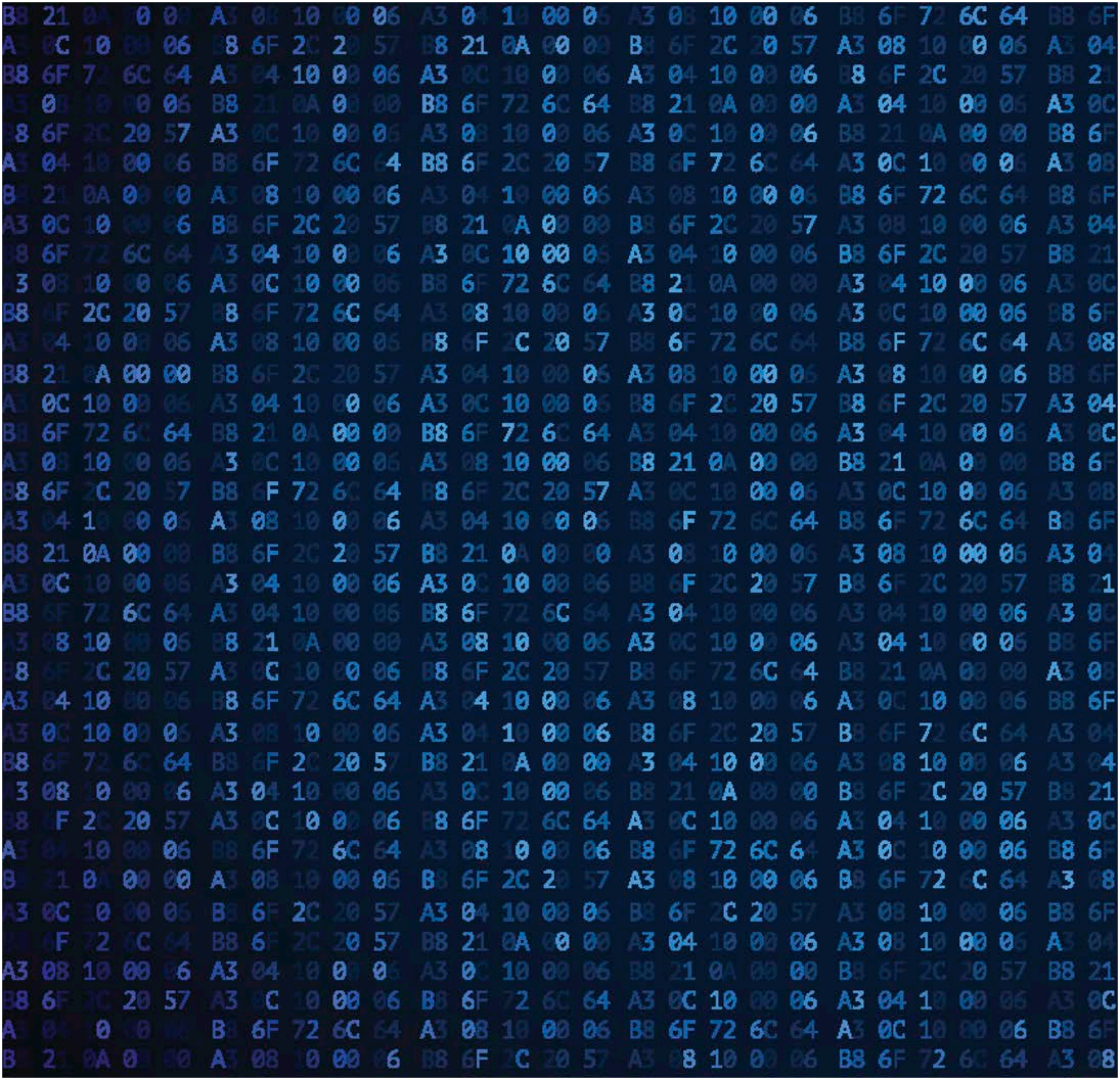
In graph limit theory, the project developed methods for studying the large-scale behaviour of bounded degree random networks and many other sparse networks. In higher order Fourier analysis, the StrucLim team researchers obtained a unified treatment of the so-called Gowers norms and Host-Kra semi-norms. "*These results have various applications. For example, we obtained a structure theory for the characteristic factors of the Host-Kra semi-norms for nilpotent actions.*" says Prof. Szegedy.

Researcher: Balázs Szegedy, Hungarian Academy of Science - Alfréd Rényi Institute of Mathematics (Hungary)

ERC project: Limits of discrete structures (StrucLim)

ERC funding: Consolidator Grant 2013, EUR 1.2 million (2014-2019)

Balázs Szegedy is a Hungarian mathematician. After finishing his PhD in 2003 he held short term positions at Microsoft Research and the Institute for Advanced Study (USA). In 2006 he joined the University of Toronto (Canada) and he returned to Hungary in 2013. Currently he works at the Alfréd Rényi Institute of Mathematics. His research focuses mainly on combinatorics and group theory.



Understanding the deformation spaces of geometric structures

Funded by the ERC, Prof. Anna Wienhard studies several new geometric forms that have been discovered over the past 20 years. These structures are closely related to the generalisation of Teichmüller space, which describes how the surface of a pretzel can be endowed with the geometry of an Escher painting.

Topologically, a surface is classified by its genus, which is determined by the number of holes in it. A sphere with no holes has genus zero, whereas a donut is genus one and a pretzel genus three. If the genus is greater than one, the uniformisation theorem states that the surface be endowed with a hyperbolic structure.

There are an infinite number of ways of endowing such a surface with a hyperbolic structure. Known as the Teichmüller space, this is a prime example of a 'deformation space' of geometric structures. In general, a deformation space describes all possible ways in which a given topological manifold can be endowed with a given geometric structure. Prof. Wienhard is investigating such deformation spaces, in particular, those related to higher Teichmüller spaces.

Higher Teichmüller spaces are connected components of the variety of representations of the fundamental group of a surface into a semi-simple Lie group, which consist entirely of discrete and faithful representations. There are two families of higher Teichmüller spaces: Hitchin components, defined for split real Lie groups; and maximal representations, defined Lie groups of Hermitian type.

Researchers recently discovered the underlying structure, which explains why there are Teichmüller spaces for some groups but not others. "*This underlying structure is a new notion of positivity in semi-simple real Lie groups, which at the same time generalises the concept of total positivity and the Lie semi-groups of Lie groups of Hermitian type*," says Prof. Wienhard.

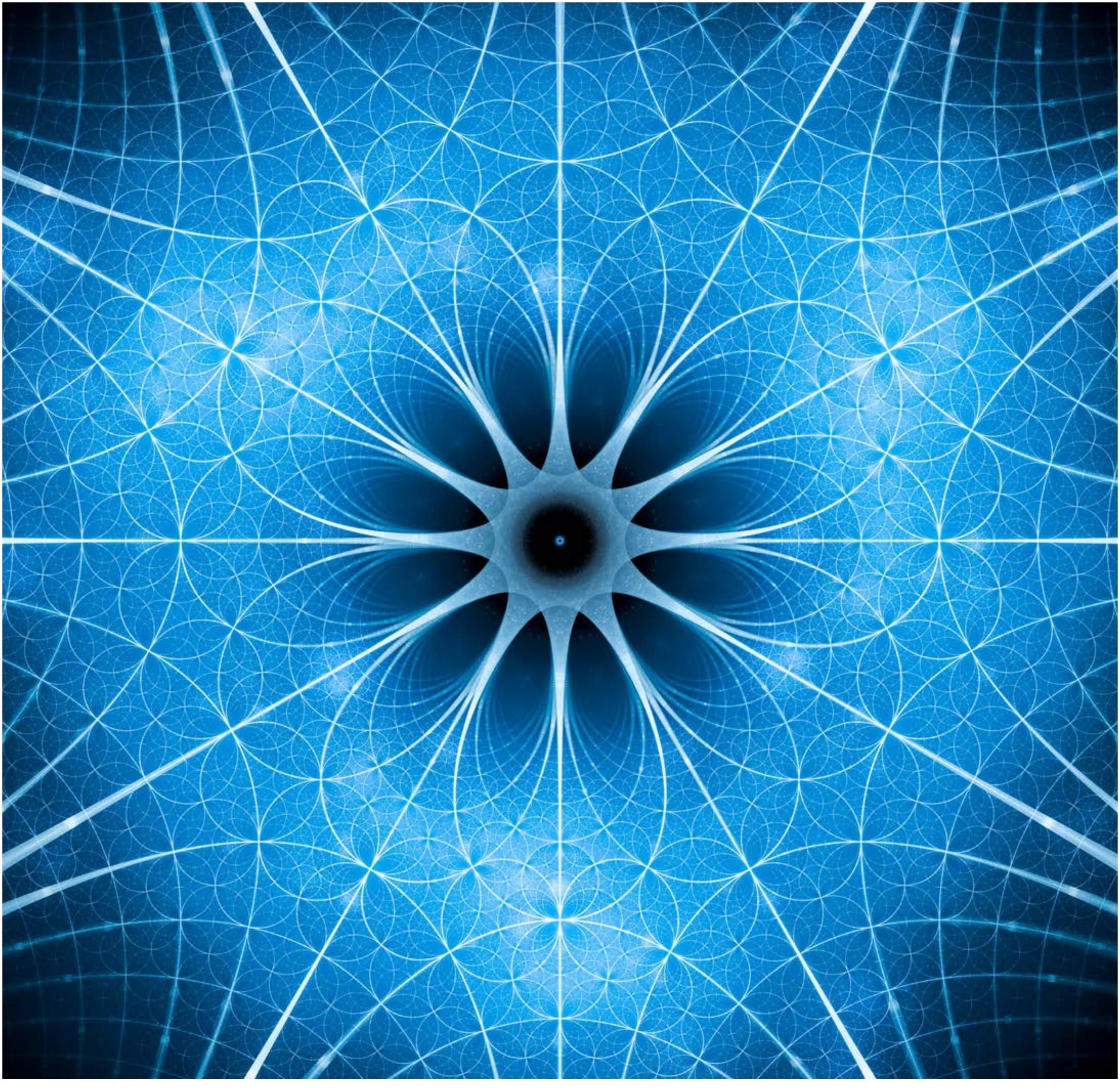
According to her, this new notion of positivity raises additional questions. "*Total positivity plays a role in many areas of mathematics and has applications in, for example, statistical mechanics*," she says. "*I think the new positivity will be a big part of this project's legacy, and I want to explore if our notion provides new approaches for applications.*"

Researcher: Anna Wienhard, Heidelberg University (Germany)

ERC project: Deformation Spaces of Geometric Structures (GEOMETRICSTRUCTURES)

ERC funding: Consolidator Grant 2013, EUR 1.6 million (2014-2018)

Anna Wienhard is a Full Professor at the Mathematical Institute of Heidelberg University, and a group leader at the Heidelberg Institute of Theoretical Studies (Germany). She is a Fellow of the American Mathematical Society. Her research is at the interface between geometry, topology and algebra. She investigates moduli spaces of geometric structures.



Statistical inference for random functions and measures

How does one infer the dynamics of a DNA minicircle in solution? How does one align the neuronal firing patterns of several neurons across individuals? These questions are intrinsically statistical, but nevertheless escape the traditional tools of statistics. The ComplexData project investigated such questions from a mathematical and an applied context.

The data revolution produces not only big data, but often complex data too: objects whose intrinsic structure requires a more sophisticated mathematical formalism than usual to analyse statistically. For instance, such objects may lie in spaces that are infinite dimensional and/or curved. The ComplexData project, funded by the ERC, developed theory and methodology for such data arising in contexts from biophysics to neuroscience.

Among its main contributions was a novel spectral framework to analyse time series of curves or surfaces – time series in Hilbert spaces - where the observation at each time point is a function itself. This was applied to provide the first rigorous statistical evidence that the base-pair composition of DNA strands influences their dynamical behaviour.

"While our work here was motivated by the dynamics of DNA in solution (seen as a curve moving in space, hence giving rise to a time series of curves), one can imagine a plethora of other situations where data fall in this framework," says Prof. Victor Panaretos.

Another of the project's key contributions was a novel framework to analyse physiological processes, such as neuronal firing patters, among several individuals where each individual has their own, and unknown, time scale. This corresponded to viewing the data as random elements of the so-called Wasserstein space, and using tools from optimal measure transport to define appropriate notions of means and common time-scales.

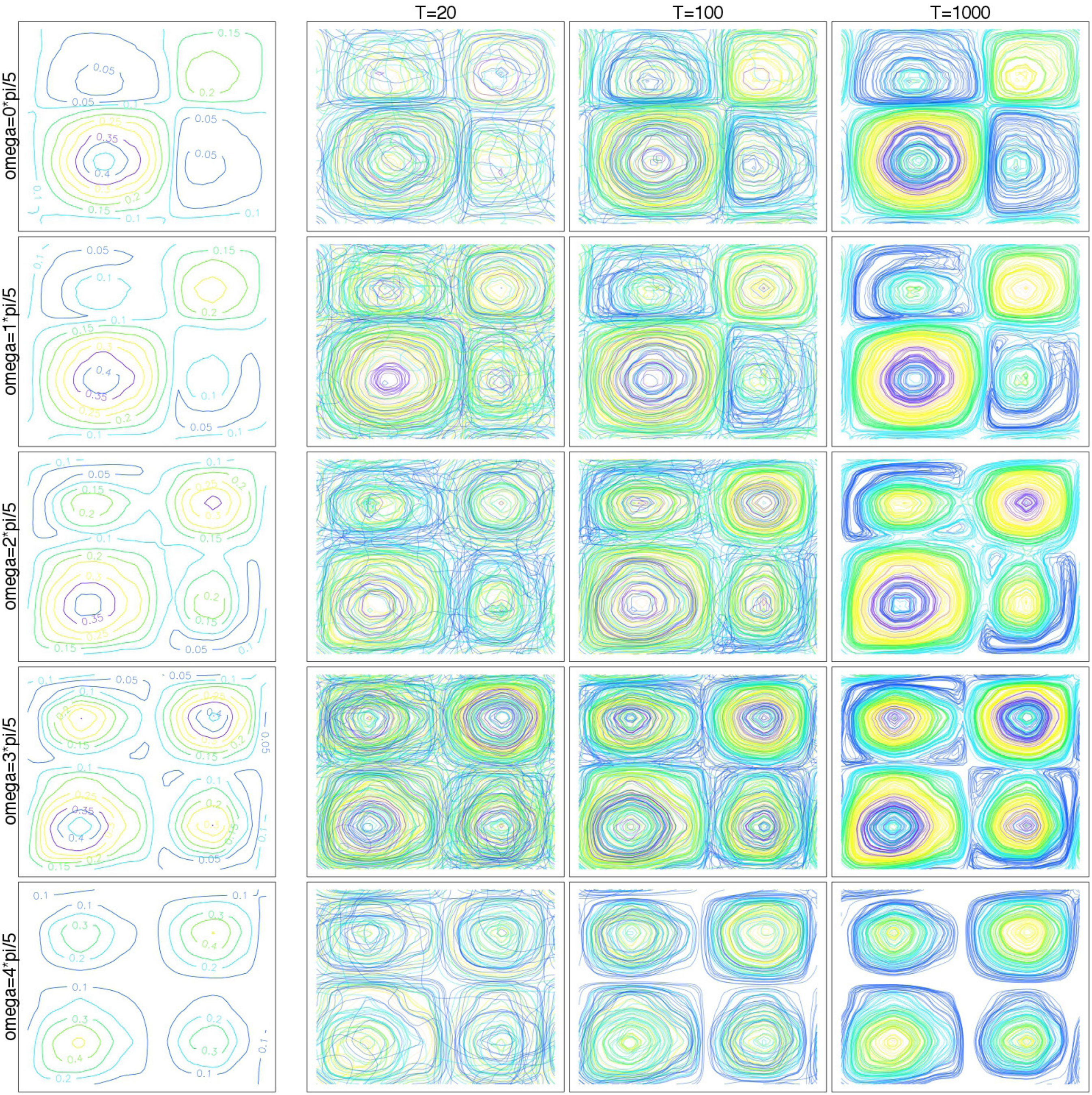
The project extended existing techniques, but also generated and applied new ones. "*We were able, for instance, to provide rigorous statistical evidence on the nature and degree of association between the base-pair composition and mechanical behaviour of DNA minicircles at persistence length*" Prof. Panaretos explains.

Researcher: Victor Panaretos, École Polytechnique Fédérale de Lausanne - EPFL (Switzerland)

ERC project: Statistics for Complex Data: Understanding Randomness, Geometry and Complexity with a view Towards Biophysics (COMPLEXDATA)

ERC funding: Starting Grant 2010, EUR 0.7 million (2011-2016)

Victor Panaretos is Associate Professor of Mathematical Statistics at EPFL (Switzerland). He obtained his PhD at UC Berkeley (USA) in 2007, receiving the Lehman Award for his thesis. His research focusses on functional and geometrical statistics. An elected fellow of the International Statistical Institute, he serves on the board of several journals, and has been named the 2019 Bernoulli Society Forum Lecturer.



Spectral density kernel amplitudes

Unravelling the mysteries of homogeneous dynamics

How do you study arithmetic objects like integer points using the theory of dynamical systems? The answer is homogenous dynamics, and this connection goes both ways. The GMODGAMMADYNAMICS project, funded by an ERC grant, took a broad approach towards studying this rich interplay.

Homogeneous dynamics is the study of the asymptotic properties of the action of Lie groups on homogenous spaces such as the space of lattices in n dimensional space. "*This project looked at the connection these actions have to number theory, to the spectral theory of homogenous spaces and to arithmetic combinatorics,*" says Prof. Elon Lindenstrauss. "*It's a beautiful area that interacts with a lot of mathematics.*"

One of the mysteries that the project team addressed was the subtle rigidity properties of higher rank abelian actions on homogenous spaces. They also focused on the combinatorial properties of arithmetic groups, equidistribution, spectral gaps, random walks and quantum ergodicity.

A key achievement of this work was the creation of a very general joining classification theorem for higher rank diagonalisable groups. "*In some sense, this tool is a definitive result: one that has already found interesting applications,*" says Prof. Lindenstrauss. For example, this joining result was used by other researchers to establish the joint distribution of integer vectors on Eucleadian spheres and the shape of the lattice orthogonal to this vector.

Another important outcome was the solving of a 50-year-old conjecture on local limit theorem for random walks on the isometry group of Euclidean spaces by a team member, Dr. Peter Varju, which gave way to a very precise local-central limit theorem for such random walks.

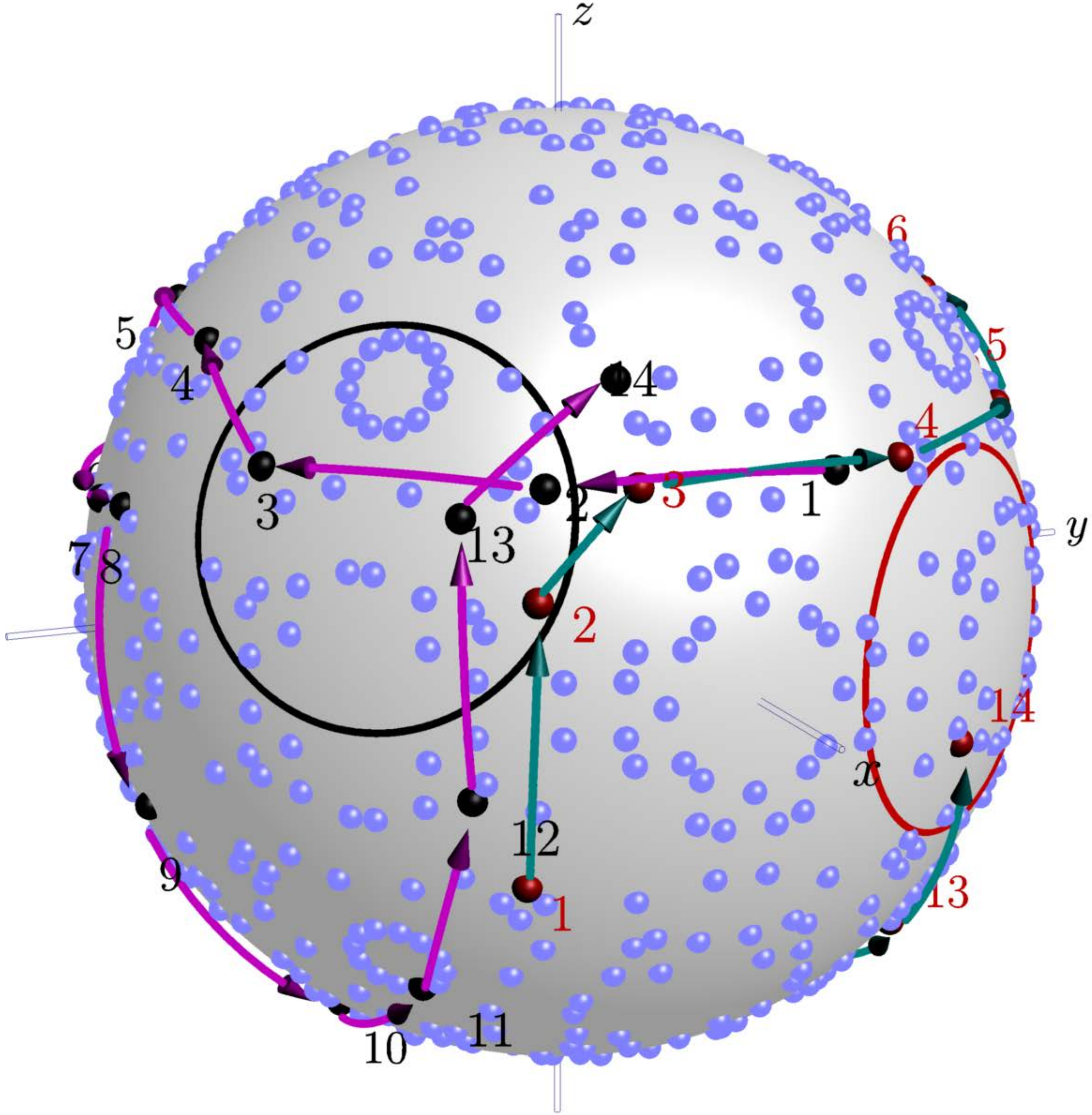
Despite these significant breakthroughs, Prof. Lindenstrauss says that this is just the beginning: "*There are ideas developed in this project that we are eager to take even further forward. Furstenberg's Conjecture on x2 x3 invariant measures and Littlewood's Conjecture are both still open, so there's still plenty more to think about!*"

Researcher: Elon Lindenstrauss, Hebrew University of Jerusalem (Israel)

ERC project: Dynamics on homogeneous spaces, spectra and arithmetic (GMODGAMMADYNAMICS)

ERC funding: Advanced Grant 2010, EUR 1.2 million (2011-2016)

Elon Lindenstrauss obtained his PhD at the Hebrew University of Jerusalem (Israel) under the guidance of Benjamin Weiss in 1999. He held positions at the IAS in Princeton, Stanford University and Princeton University (USA) before returning to the Hebrew University in 2008 where he is currently Professor of Mathematics and since 2016 the Chair of the Einstein Institute of Mathematics. He received several prizes for his mathematical achievements, including a Fields Medal. He is a member of the Israel Academy of Sciences and Humanities and the Academia Europaea.



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Jean-Pierre Bourguignon
ERC President and Chair of its Scientific Council

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